Energy-Efficient and Delay-Constrained Broadcast in Time-Varying Energy-Demand Graphs

Chenxi Qiu, Haiying Shen and Lei Yu
Dept. of Electrical and Computer Engineering
Clemson University, Clemson, USA
$\{$*chenxiq, shenh, leiy*$\}$*@clemson.edu*

***Abstract*—In this paper, we study the *minimum energy broadcast problem* in time-varying graphs (TVGs), which are a very useful high level abstraction for studying highly dynamic wireless networks. To this end, we first incorporate a channel model, called *energy-demand functions*, to the current TVGs, namely *time-varying energy-demand graphs* (*TVEGs*). Based on this model, we formulate the problem: given a TVEG, what is the optimal schedule (*i.e.*, which nodes should forward a packet in what times and at what power levels) to minimize the energy consumption of the broadcast? We prove the problem to be NP-hard and $o(\log N)$ inapproximable. It is a challenge to find a solution for this problem on continuous time. Fortunately, we prove that the problem on continuous time is equivalent to the problem on certain discrete time points, called *discrete time set (DTS)*. Based on this property, we propose polynomial time solutions for this problem with different channel models, and evaluate the performance of these methods from real-life contact traces.**

***Keywords*-time-varying graphs; NP-hard; discrete time points;**

I. Introduction

In wireless networks, broadcast is particularly an important mechanism for disseminating a packet from one source node to all other nodes in the network. Among numerous challenges confronted in designing protocols for broadcast, the energy problem stands out as one of the most critical issues. Over the past years, many works have studied the minimum-energy broadcast problem in various types of wireless networks, such as ad hoc networks [1]–[4] and cooperative networks [5], [6]. However, all of these works simply assume that the network topology never changes, which is not valid for highly dynamic networks, where network topology and link channel condition change over time. For example, in mobile ad hoc networks, the network topology changes dramatically over time due to node mobility, while in sensor networks, links only exist when two neighbouring sensors are awake and have power. Obviously, the previous broadcast protocols designed for static networks are not applicable to dynamic networks.

Recently, a theoretical framework called *time-varying graphs* (TVGs) [7] (or *temporal networks*, *evolving graphs*) has been developed for the study of highly dynamic networks [8], [9]. Different from traditional static graphs, where edges and vertices never change over time, in time-varying graphs, vertices and edges appear and disappear as a function of time. Hence, the conclusions from static graphs may not hold true in time-varying graphs. Thus, some typical problems (such as *shortest path* (or *journey*) [8], *reachability* [10], and *random walk* [11]) in static networks have been re-studied in time-varying graphs. However, there is no previous work that has studied the *minimum-energy broadcast problem* based on the TVG model. It is because that the existing time-varying graphs cannot reflect the channel conditions of each link, which is an essential information for studying energy problem in dynamic networks.

In this paper, our objective is to design an energy-efficient time-constrained broadcast scheme for dynamic networks. We first build a model called *time-varying energy-demand graphs* (*TVEGs*) which reflects the channel condition of the edges. Based on this model, we then formulate and analyze the problem: given a TVEG, what is the optimal relay allocation schedule (*i.e.*, specifying relay nodes to forward a packet at certain times and using certain levels of energy) to minimize the energy consumption of the broadcast? Finally, we propose several polynomial time solutions for the problem for different channel models.

A TVEG is built based on current time-varying graphs by adding the channel condition to each edge in the graph. In our model, we consider fading channels, where transmissions between nodes are susceptible to random fluctuations in signal strength. For any single transmission, the more energy used by the transmitter, the higher probability that the packet can be decoded by the receiver. Hence, given an edge and a time point, we use a probabilistic model called *energy-demand (ED) function* to describe the relationship between the energy cost and the probability of successful transmission at this time. As a result, TVEG reflects both topology and link channels of dynamic networks, which can be used to study the minimum-energy broadcast problem.

Based on TVEGs, we formulate the relay allocation optimization problem as *Time-varying Minimum-Energy Delay-Constrained Broadcast problem* (*TMEDB*). This problem's objective is to find a relay allocation schedule to minimize the energy consumption of the broadcast while guaranteeing that i) the broadcast can be completed within a time constraint, and ii) the delivery ratio is higher than a threshold. We prove this problem is NP hard and $o(\log N)$ inapproximable, where N is the number of nodes in the network.

It is a challenge to solve TMEDB defined on continuous time. Fortunately, we prove that it is equivalent to TMEDB defined on some specified discrete time points, called *discrete time set* (*DTS*). Based on the existing method for *minimum-energy multicast tree problem* [3], we propose polynomial time solutions for TMEDB with different channel conditions. Specifically, we build an auxiliary graph for TMEDB with static channel and present an approximate algorithm, which delivers an approximate solution with a bounded performance guarantee $O(N^{\epsilon})$, where ϵ $(0 < \epsilon \leq 1)$ is a constant. For TMEDB with fading channels, we first find the relay nodes and their packet transmission times, and then model the energy allocation problem as a *non-linear programming problem*. We summarize the contributions of this paper as follows:

- *Time-varying energy-demand graphs*. TVEGs enhance the existing time-varying graphs by mapping each edge to an energy-demand function to reflect the change of each link's channel in the network, which enables to study the minimum-energy broadcast problem.
- *Problem formulation and analysis*. Based on TVEGs, we formulate the TMEDB problem. The objective of this problem is to find a relay allocation schedule to minimize the energy consumption of the broadcast while guaranteeing that i) the broadcast can be completed within a time constraint; and ii) the delivery ratio is higher than a threshold. In addition, we prove this problem is NP-hard and $o(\log N)$ inapproximable.
- *Discrete time set*. It is a challenge to find a solution for this problem on continuous time. Fortunately, we prove that it is equivalent to the problem on DTS.
- *Solutions to the formulated problem*. To solve this problem, we first prove that TMEDB defined on continuous time is equivalent to the problem defined on some specified discrete time points, called *discrete time sets* (*DTS*).

We present an approximate algorithm for TMEDB with static channel. For the fading channel case, we model the energy allocation problem as a *non-linear programming problem*. Finally, we evaluate the performance of these methods from real-life contact trace in the Huggle project [12].

The remainder of this paper is organized as follows. Section II presents related work. Section III builds the mathematical model of TVEGs. Section IV defines the TMEDB problem and proves the hardness of this problem. Section V proves that TMEDB defined on continuous time is equivalent to TMEDB defined on DTS. Guided by these properties, in Section VI, we propose schemes for TMEDB in both static channel and fading channels. Section VII evaluates the performance of our proposed schemes in comparison with other schemes. Section VIII concludes this paper with remarks on our future work.

II. Related Work

Minimum-energy broadcast. The problem of minimizing the energy consumption of broadcast in wireless networks has received significant attention over the last few years [1]–[6]. Čagalj *et al.* [1] and Li *et al.* [4] considered the minimum-energy broadcast problem in all-wireless networks, where all nodes are linked via short-range ad hoc radio connections and the communication is supported by multi-hop transmissions. Both works proved the NP-hardness of the problem in general case, and the work in [1] further proved that the problem remains NP-hard in two-dimensional Euclidean metric space. Liang [3] assumed a different kind of wireless network, in which the transmitting power of each node is finitely adjustable. The author proved the NP-hardness of the problem and presented an approximation algorithm by reducing the problem to an optimization problem on an auxiliary weighted graph. Ashwinder *et al.* [2] addressed the minimum-energy broadcast problem in ad hoc wireless networks and proposed a distributed algorithm that computes all possible broadcast trees simultaneously with $O(N^2)$ message complexity. Maric *et al.* [5] and Hong *et al.* [6] proved that the energy-efficient broadcast problem is NP-hard in cooperative communication, where each packet receiver can cooperatively combines received weak signals from different senders to recover the original packet. Hermann *et al.* [13] showed that optimal broadcast solutions in non-fading environments may suffer a low delivery ratio in fading environments. Then, they reformulated the problem by incorporating fading channel models to broadcast backbone construction. They proved that the problem is still NP-hard, and proposed a heuristic algorithm, which uses probability of successful communication as a metric to select relay nodes. Though these previous works have discussed the minimum-energy broadcast problems in various scenarios, to the best of our knowledge, none of them studied the problem in dynamic networks, in which the network topology and channel conditions of each link change over time.

Time-varying graphs. Time-varying graphs are a very useful high-level abstraction for studying topology over time in highly dynamic networks. Many of the normal concepts of static graphs have no obvious counterpart in time-varying graphs. Hence, a number of important concepts in static graphs have been redefined in TVGs, e.g, *temporal distance*, *journey* (*path*) [8], and *reachability* [9]. Casteigts *et al.* [7] integrated the collection of these concepts into a unified coherent framework, and first formally defined TVGs. Furthermore, some typical problems (e.g., reachability and broadcast) which have been well-understood in static graphs need be re-studied in time-varying graphs. For example, Whitbeck *et al.* [10] introduced *temporal reachability graphs* to record the connectivity of any pair of nodes during any time interval in TVGs, and proposed a time efficient algorithm for computing these graphs. Daniel Figueiredo *et al.* [11] studied the behavior of a continuous time random walk on a stationary and ergodic TVG, and characterized the stationary distribution of the walker in some specified cases. However, as far as we know, there is no previous work that has studied the minimum-energy broadcast problem using the TVG theoretical framework, since existing TVGs cannot reflect the channel conditions of each link.

III. Time-varying Energy-demand Graphs

In this section, we first introduce TVGs (Section III-A). Based on TVGs, we build the mathematical model of TVEG-s (Section III-B), which maps each edge in a TVG to an energy-demand function that reflects the channel conditions of edges (i.e., links). Finally, we present energy-demand functions based on different channel models (Section III-C). TVEGs enable us to study the minimum-energy broadcast problem in dynamic networks, which will be presented in Section IV.

A. Time-Varying Graphs

We introduce TVGs formally defined in [7] along with definitions and notations. Consider a finite set of nodes V and a set of possible edges $E \subseteq V \times V$. Events occur over a time span $\mathcal{T} \subseteq \mathbb{T}$, where $\mathbb{T}$ is the temporal domain ($\mathbb{T}$=$\mathbb{N}$ for discrete time systems and $\mathbb{T} = \mathbb{R}^+$ for continuous time systems). In the general case, a TVG is a tuple $\mathcal{G} = (V, E, \mathcal{T}, \rho, \zeta)$. ρ: $E \times \mathcal{T} \to [0,\ 1]$ is called *presence function*, which indicates the probability that a given edge exists at a given time. $\zeta : E \times \mathcal{T} \to \mathbb{T}$ is called *latency function*, which indicates the latency to traverse a given edge at a given time and the latency may vary at different times.

Since continuous time systems are more practical and complex than the discrete time systems, in this paper, we consider a continuous time TVG and assume a constant ζ function such that $\forall (e,t) \in E \times \mathcal{T}$, $\zeta(e,t) = \tau$ ($\tau \geq 0$) is our uniform edge traversal time. A TVG is *deterministic* if ρ: $E \times \mathcal{T} \to \{0,1\}$; otherwise is *non-deterministic*. In this paper, we only consider deterministic TVGs. We denote the edge connecting two vertices v_i and v_j ($v_i, v_j \in V$) by e_{v_i,v_j} or $e_{i,j}$. Also, we use $\mathbb{R}^*$ to denote $\mathbb{R}^+ \cup 0$.

Definition 3.1: (*Journey*) [7] A journey in $\mathcal{G}$ is a sequence of couples $\mathcal{J} = \{(e_{i_1,j_1}, t_1), ..., (e_{i_k,j_k}, t_k)\}$ such that $\forall l < k$: (i) $j_l = i_{l+1}$; (ii) $\forall t$ *s.t.* $t_l \leq t < t_l + \tau$, $\rho(e_{i_l,j_l}, t) = 1$; and (iii) $t_{l+1} \geq t_l + \tau$.

Here, $|\mathcal{J}| = k$ is journey $\mathcal{J}$'s topological length (*i.e.*, the number of hops). A journey $\mathcal{J} = \{(e_{i_1,j_1}, t_1), ..., (e_{i_k,j_k}, t_k)\}$ is called *non-stop journey* if $\forall l < k$, $t_{l+1} = t_l + \tau$. We say v_i is in $\mathcal{J}$ if $\exists (e_{i_l,j_l}, t_l) \in \mathcal{J}$ such that v_i is a vertex of e_{i_l,j_l}. We say a journey has no *circle* if it has no repeated node. In the following, we only consider the journey without circle. For any pair of nodes v_i and v_j in a journey $\mathcal{J}$, we say v_i is *precede* to v_j or v_j is *succeeding* to v_i (denoted by $v_i \prec_{\mathcal{J}} v_j$), if $\mathcal{J}$ arrives at v_i before v_j. In addition, $v_i \preceq_{\mathcal{J}} v_j$ means $v_i \prec_{\mathcal{J}} v_j$ or $v_i = v_j$. Finally, $\mathsf{departure}(\mathcal{J})$ and $\mathsf{arrival}(\mathcal{J})$ denote the starting time t_1 and the ending time $t_k + \tau$ of $\mathcal{J}$, respectively. Table I lists the main notations used in this paper.

Table I
Notations and definition

Notation	Description	Notation	Description
V	Node set	v_i	The i^{th} node;
E	Edge set	ρ	Presence function
$\mathcal{J}$	Journey	τ	Edge traversal time
φ	ED-function	$\mathbf{S}$	Broadcast relay schedule
$\mathbf{W}$	Cost vector	$\mathbf{R}$	Relay node vector
$\mathbf{T}$	Time vector	$\mathbf{P}_i^{\text{ad}}$	Adjacent partition of v_i
$\mathbf{D}_V$	DTS on V	$\mathbf{P}_i^{\text{st}}$	Status partition of v_i
$e_{i,j}$ or e_{v_i,v_j}	Edge between v_i and v_j	$\mathbf{P}_i^{\text{di}}$	Discrete time partition of v_i

B. Time-Varying Energy-Demand Graphs

In this section, we introduce how to build a TVEG by embedding *energy-demand (ED) functions* to each edge of the existing TVG to reflect the link channels. We assume a continuous energy cost (or cost in short) set $\mathcal{W}$ that each node can use, which has lower bound $w_{\min}$ and higher bound $w_{\max}$. Given $\mathcal{G} = (V, E, \mathcal{T}, \mathcal{F}, \rho, \zeta)$, for each edge $e_{i,j} \in E$, we use an ED-function $\varphi_t^{e_{i,j}}$: $\mathcal{W} \to \mathbb{R}^*$ to reflect the relationship between the cost of v_i (the sender) and the probability of failure transmission to v_j (the receiver) at time t. We use $\mathcal{F} = \{\varphi_t^e, e \in E, t \in \mathcal{T}\}$ to represent the set of all ED-functions, called *ED-function class*. ED-functions have the following properties:

Property 3.1: (*ED-function*) For each $\varphi_t^e \in \mathcal{F}$
(i) If $\rho(e,t) = 1$ and $w_{\max} \to \infty$, $\lim_{w \to \infty} \varphi_t^e(w) = 0$;
(ii) If $\rho(e,t) = 1$ and $w_{\min} = 0$, $\varphi_t^e(0) = 1$;
(iii) If $\rho(e,t) = 0$, $\varphi_t^e(w) = 1, \forall w \in \mathcal{W}$;
(iv) $\varphi_t^e(w)$ is non-increasing.

Then, we formally define TVEG as follows:

Definition 3.2: (*Time-varying energy-demand graph*) Given an ED-function class $\mathcal{F}$ and a TVG $\mathcal{G}$, TVEG $\mathcal{G}_{\mathcal{F}}$ is extended from $\mathcal{G}$ by embedding an ED-function in $\mathcal{F}$ to each edge in E. In general, $\mathcal{G}_{\mathcal{F}}$ can be represented by a tuple $\mathcal{G}_{\mathcal{F}} = (V, E, \mathcal{T}, \mathcal{F}, \rho, \zeta, \psi)$, where cost function ψ: $E \times \mathcal{T} \to \mathcal{F}$ indicates which ED-function is embedded to an edge at a given time.

C. ED-functions for Different Channel Models

To build a TVEG, we need to embed each edge in TVG with an ED-function, which is determined by the channel model of the network. In this section, we demonstrate how to model the ED-function from the *step function* for static channel [14], and from the *Rayleigh fading ED-function* for fading channel model [13]. We call the two ED-functions *step ED-function* and *Rayleigh ED-function*, respectively.

Step ED-function. For a single transmission, whether or not a packet is successfully received depends on the instantaneous SNR equals $S_{i,j,t}/N_0$ at the receiver, where N_0 is the noise power density and $S_{i,j,t}$ is the signal power received at v_j from v_i at time t. In a static channel environment [14], given time t, the radio propagation gain from v_i to v_j is modeled as a constant $h_{i,j,t}$, so SNR equals $w \times h_{i,j,t}/N_0$, where w is the cost of v_i. The necessary and sufficient condition for successful decoding at v_j is

$$\frac{w \times h_{i,j,t}}{N_0} \geq \gamma_{\text{th}} \tag{1}$$

where γ_{th} is the fixed decoding threshold. From Formula (1), we can derive the ED-function $\varphi_t^{e_{i,j}}$ as follows

$$\varphi_t^{e_{i,j}}(w) = \begin{cases} 0 & \text{if } \frac{N_0\gamma_{\text{th}}}{h_{i,j,t}} \leq w \leq w_{\max} \\ 1 & \text{if } w_{\min} \leq w < \frac{N_0\gamma_{\text{th}}}{h_{i,j,t}} \end{cases} \tag{2}$$

It implies that v_j can successfully decode a packet from v_j at time t iff the cost of v_i is no less than $\frac{N_0\gamma_{\text{th}}}{h_{i,j,t}}$. We call $\frac{N_0\gamma_{\text{th}}}{h_{i,j,t}}$ the *minimum cost* of v_i to v_j.

Rayleigh fading ED-function. In the Rayleigh fading model with a frequency-flat time-varying wireless channel [13], for the transmitted signal received by v_j from sender v_i, the channel effect can be modeled by a single, complex, random channel coefficient $h_{i,j,t}$. As [13], we consider a Rayleigh fading channel in which all $|h_{i,j,t}|^2$ are independent and exponentially distributed with a mean value

$$\sigma_{i,j,t}^2 = w d_{i,j,t}^{-\alpha} \tag{3}$$

where w, $d_{i,j,t}$ and α represent the transmission power of v_i at time t, the distance between v_i and v_j at time t and the path loss exponent. The instantaneous signal power $S_{i,j,t}$ received at v_j from v_i at time t is a random variable with Cumulative Distribution Function (CDF)

$$F_{S_{i,j,t}} = 1 - \exp\left(-\frac{x}{\sigma_{i,j,t}^2}\right). \tag{4}$$

We use a non-negative random variable $X_{i,j,t} = S_{i,j,t}/N_0$ to represent the SNR transmitted from v_i to v_j at time t ($X_{i,j,t} \sim X_{j,i,t}$), and then v_j can successfully receive a packet iff $X_{i,j,t} \geq \gamma_{\text{th}}$. $X_{i,j,t}$ has CDF $F_{X_{i,j,t}} = 1 - \exp(-N_0 x/\sigma_{i,j,t}^2)$. Then, the probability of the packet that failes to be received by v_j from v_i at time t equals $1 - \exp(-N_0\gamma_{\text{th}}/\sigma_{i,j,t}^2)$ or $1 - \exp(-N_0\gamma_{\text{th}}/d_{i,j,t}^{-\alpha} w)$. Accordingly, the Rayleigh fading ED-function $\varphi_t^{e_{i,j}}(w)$ can be derived as follows:

$$\varphi_t^{e_{i,j}}(w) = 1 - \exp\left(-\frac{\beta_{i,j,t}}{w}\right) \tag{5}$$

where $\beta_{i,j,t} = N_0\gamma_{\text{th}}/d_{i,j,t}^{-\alpha}$ and $w \in [w_{\min}, w_{\max}]$.

IV. The Time-varying Minimum-Energy Delay-Constrained Broadcast Problem

Now that we have built the mathematical model for studying energy problem in highly dynamic networks, in this section we formally formulate the problem based on this problem, namely *time-varying minimum-energy delay-constrained broadcast* (TMEDB) problem, and analyze the hardness of the problem.

Given a TVEG $\mathcal{G}_{\mathcal{F}}$ and a source node v_{s}, *broadcast relay schedule* (or simply *schedule*) determines which nodes should be selected as relay nodes at what times and at what power should communication take place. Suppose a schedule contains totally n transmissions, then this schedule can be represented by an $n \times 3$ non-homogenous matrix $\mathbf{S} = [\mathbf{R}, \mathbf{T}, \mathbf{W}]$, where $\mathbf{R} \in V^n$ (called relay node vector) records each transmission's relay node, $\mathbf{T} \in \mathcal{T}^n$ (called time vector) records each transmission's time, and $\mathbf{W} \in \mathcal{W}^n$ (called cost vector) records each transmission's cost. We use r_k, t_k and w_k to represent the k^{th} element in $\mathbf{R}$, $\mathbf{T}$, and $\mathbf{W}$, respectively. Then, $\mathbf{S}$ can be also represented by $[s_1, ..., s_n]^{\text{T}}$, where each element $s_k = [r_k, t_k, w_k]$ describes the information of the k^{th} transmission in the schedule (*i.e.*, relay node, time, and power level). We define the cost of $\mathbf{S}$ as the total cost of all its transmissions, *i.e.*, $\sum_{k=1}^n w_k$. Note that a relay node may forward a packet multiple times in a schedule, which implies that a node can be repeated in $\mathbf{R}$.

For each $v_i \in V$, let $p_{i,t}$ (or $p_{v_i,t}$) denote the probability that v_i cannot successfully receive the packet by time t. Obviously, $\forall t_1, t_2 \in \mathcal{T}$, if $t_1 \leq t_2$, then $p_{i,t_1} \geq p_{i,t_2}$. We say v_i has been *informed* by t iff $p_{i,t} \leq \varepsilon$, where ε is the error acceptable rate; otherwise, we say v_i is still *uninformed* by t. At any time, the status of a node is either "informed" or "uninformed".

Suppose a packet is transmitted from v_i to v_j at time t, to complete this transmission, it requires $\rho(e_{i,j}, t') = 1$, $\forall t' \in [t, t+\tau]$; that is, there is a link between v_i and v_j at time t'. We use $\rho_\tau(e_{i,j}, t)$ to denote whether v_i and v_j are connected during $[t,\ t+\tau]$:

$$\rho_\tau(e_{i,j}, t) = \begin{cases} 1 & \text{if } \rho(e_{i,j}, t') = 1,\ \forall t' \in [t,\ t+\tau] \\ 0 & \text{otherwise} \end{cases}$$

In this paper, we assume that τ is small enough so that for any pair of nodes v_i and v_j, cost $\varphi_t^{e_{i,j}}$ is unchanged during $[t, t+\tau]$, $\forall \rho_\tau(e_{i,j}, t) = 1$. We say v_i is *adjacent* to v_j at time t iff v_i can complete a transmission to v_j at time t, *i.e.*, $\rho_\tau(e_{i,j}, t) = 1$.

For each node $v_i \in V$, v_i is uninformed by time t iff there is no successful transmission to v_i before t.

[1] In these fading models 1. σ^2 is the expected value of r^2; v^2 is the power of the Line-of-Sight (LOS) signal component; k is the fading figure (degrees of freedom related to the number of added Gaussian random variables) [15]. 2. $I_0(\cdot)$ is the modified Bessel function of the first kind with order zero; $\Gamma(a,b) = \int_b^\infty t^{a-1}e^{-t}\mathrm{d}t$, and $\gamma(a,b) = \int_0^b t^{a-1}e^{-t}\mathrm{d}t$ [16].

[2] When $w = 0$, ED-function $\varphi(w) = 1$.

Then, given a broadcast schedule $\mathbf{S} = [s_1, ..., s_n]^{\mathrm{T}}$, where $s_k = [r_k, t_k, w_k]$ $(1 \leq s_k \leq n)$, the probability that v_i is uninformed by time t can be calculated by

$$p_{i,t} = \prod_{t_k \leq t, s_k \in \mathbf{S}, \rho_\tau(e_{r_k,v_i}, t_k)=1} \left(\varphi_{t_k}^{e_{r_k,v_i}}(w_k) \right). \quad (6)$$

The objective of the TMEDB problem is to find a schedule that takes the minimum energy cost such that all the nodes can be informed. Formally, the decision version of TMEDB is defined as follows:

Instance: A finite set of nodes $V = \{v_1, ..., v_N\}$, a source node $v_s \in V$, a set of edges $E \subseteq V \times V$, a cost set $\mathcal{W}$, a time domain $\mathcal{T}$, a ED-function class $\mathcal{F}$, a presence function ρ, a latency function ζ, a cost function ψ, and three constants C, T, and ε.

Question: Existence of a schedule $\mathbf{S} = [\mathbf{R}, \mathbf{T}, \mathbf{W}]$ such that the following conditions are satisfied:

(i) All relay nodes have been informed by they forward the packet: $p_{r_k,t_k} \leq \varepsilon$, $\forall r_k$ in $\mathbf{R}$;

(ii) All nodes will eventually be informed by time T: $\exists t \leq T - \tau$ such that $p_{i,t} \leq \varepsilon$, $\forall v_i \in V$;

(iii) Broadcast latency is no larger than T: $\max\{t_1, ..., t_n\} + \tau \leq T$;

(iv) The cost of $\mathbf{S}$ is no larger than C: $\sum_{k=1}^{n} w_k \leq C$.

We say a schedule is *feasible* if the schedule satisfies the above four conditions. In the following we analyze the complexity of TMEDB (Theorem 4.1, Corollary 4.1 and Corollary 4.2). Due to space constraints, detailed proofs of the theorem and corollaries are omitted here and can be found in our technical report [17].

Theorem 4.1: TMEDB is NP-hard and it remains NP-hard if restricted on the ED-function class $\mathcal{F}$ and cost set $\mathcal{W}$ that satisfy the condition in which for all ε, there exists $\varphi \in \mathcal{F}$ and $w \in \mathcal{W}$ such that $\varphi(w^-) \geq 1 - \varepsilon$ and $\varphi(w) < 1 - \varepsilon$.

Proof: Due to space constraints, detailed proofs of above results are omitted here. The basic idea is to construct a polynomial time reduction of the NP-hard problem of Set Covering [18] to TMEDB with the restriction. That is, for any instance in Set Covering, we can always construct a TMEDB instance such that a solution exists for the Set Covering instance iff there exists a feasible schedule for the TMEDB instance. After proving TMEDB with the restriction is NP-hard, NP-hardness of the unrestricted problem follows immediately by restriction. ■

Corollary 4.1: TMEDB is $o(\log N)$ inapproximable and remains $o(\log N)$ inapproximable with the restrictions in Theorem 4.1.

Proof: The reduction used in constructing the instance in Theorem 4.1 preserves the approximation factor. That is, if one can find an α-approximation for TMEDB given the above restrictions, by extension, there must exist an α-approximation for Set Cover. Because the Set Cover problem is $o(\log N)$ inapproximable [18], thus TMEDB with the restrictions must be $o(\log N)$ inapproximable. ■

Corollary 4.2: TMEDB remains NP-hard and $o(\log N)$ inapproximable if ED-function class $\mathcal{F}$ is a class of

(i) step ED-functions (Formula (2));

(ii) Rayleigh fading ED-functions (Formula (5)).

Proof: (i) Given v_i, v_j, and t, and constants w, γ_{th}, and ε, according to Formula (2), if $N_0\gamma_{\mathrm{th}}/w_{\max} \leq h_{i,j,t} \leq N_0\gamma_{\mathrm{th}}/w_{\min}$, there exists cost $w = N_0\gamma_{\mathrm{th}}/h_{i,j,t}$ $(w \in \mathcal{W})$ such that $\varphi_t^{(v_i,v_j)}(w) \leq \varepsilon$ and $\varphi_t^{(v_i,v_j)}(w^-) > \varepsilon$.

(ii) Given a pair of nodes v_i, v_j, time t, and constants w, α, γ_{th}, and ε, according to Formula (5), if

$$\left(\frac{N_0\gamma_{\mathrm{th}}}{w_{\min} \ln(1/\varepsilon)} \right)^{\frac{1}{\alpha}} \leq d_{i,j,t} \leq \left(\frac{N_0\gamma_{\mathrm{th}}}{w_{\max} \ln(1/\varepsilon)} \right)^{\frac{1}{\alpha}}, \quad (7)$$

there exists cost $w = \frac{N_0\gamma_{\mathrm{th}}}{\ln \frac{1}{\varepsilon} d_{i,j,t}^{-\alpha}}$ $(w \in \mathcal{W})$ such that $\varphi_t^{(v_i,v_j)}(w) \leq \varepsilon$ and $\varphi_t^{(v_i,v_j)}(w^-) > \varepsilon$. Based on Theorem 4.1, the proof completes. ■

V. Discrete Time Set

The continuous time systems are more complex than the discrete time systems, which poses a formidable challenge to determine the transmission times in broadcast relay schedule for TMEDB (which is defined on continuous time). In this section, we first find the discrete transmission time series, called discrete time set (DTS), that still makes a feasible schedule $\mathbf{S}$ for TMEDB feasible. We then prove that finding the optimal schedule on continuous time is equivalent to finding it on a DTS.

Assume there exists a feasible schedule $\mathbf{S}$ for the TMEDB problem. Recall $\mathbf{S}$ can be represented by $[s_1, ..., s_n]^{\mathrm{T}}$, where each element $s_k = [r_k, t_k, w_k]$ represents a single transmission in which node r_k is scheduled to transmit the packet at t_k using cost w_k. Then, r_k must be informed and connected with a set of nodes at t_k. We aim to find the earliest transmission time point (denoted by $\hat{t}_k$) for each t_k that makes $\mathbf{S}$ still feasible. This means that at time $\hat{t}_k$, node r_k still has the same set of connected nodes and the same status (informed or uninformed) as those at time t_k. Therefore, we can find the time interval series, called *adjacent partition* of node v_i, such that the set of v_i's connected nodes are unchanged in each interval. We also find the time interval series, named *status partition* of v_i, such that v_i's status is unchanged in each interval. By combining these two time interval series, we then generate a new time interval series (called DTS); in each interval, the set of v_i's connected nodes and v_i's status are unchanged. Below, we introduce how to generate *adjacent partition*, *status partition*, and DTS, respectively.

Definition 5.1: (*Partition*) A partition $\mathbf{P}$ of the time span $\mathcal{T}$ is a finite sequence of time points in the form of

$$0 = t_0 < t_1 < t_2 < ... < t_{m-1} < t_m = T.$$

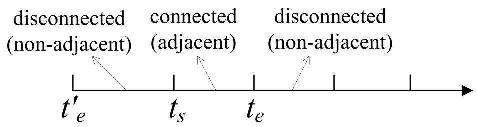


Figure 1. Adjacent interval and non-adjacent time interval.

We say $[t_k,\ t_{k+1})$ $(1 \leq k \leq m{-}1)$ is a time interval (interval in short) in the partition $\mathbf{P}$. Given $\mathcal{T}$'s three partitions: $\mathbf{P}_1$, $\mathbf{P}_2$, and $\mathbf{P}_3$, $\mathbf{P}_3$ is called the *combination* of $\mathbf{P}_1$ and $\mathbf{P}_2$ (*i.e.*, $\mathbf{P}_3 = \mathbf{P}_1 \cup \mathbf{P}_2$) if $\mathbf{P}_3$ is composed of ordered time points from both $\mathbf{P}_1$ and $\mathbf{P}_2$. In general,

$$\mathbf{P}_{n+1} \triangleq \bigcup_{k=1}^{n} \mathbf{P}_k \tag{8}$$

if $\mathbf{P}_{n+1}$ is the combination of $\mathbf{P}_1$, ..., $\mathbf{P}_n$, where $\mathbf{P}_1$, ..., $\mathbf{P}_{n+1}$ are all partitions of $\mathcal{T}$.

Adjacent partition. As shown in Fig. 1, the link between a pair of nodes v_i and v_j appear and disappear in the time-varying graph over time; that is, two nodes are connected and disconnected in the dynamic wireless network over time. We call the time interval that two nodes are connected (*i.e.*, adjacent in the graph) their *adjacent interval*, and call the time interval that two nodes are disconnected (*i.e.*, non-adjacent in the graph) their *non-adjacent interval*. Then, the time span $\mathcal{T}$ of v_i to v_j can be partitioned into a sequence of intervals, denoted by $\mathbf{P}^{\text{ad}}_{i,j}$; each interval is either their adjacent interval or their non-adjacent interval. Further, we define the adjacent partition of v_i as

$$\mathbf{P}^{\text{ad}}_{i} \triangleq \bigcup_{v_j \in V/v_i} \mathbf{P}^{\text{ad}}_{i,j} \tag{9}$$

Obviously, the set of v_i's connected nodes are unchanged in each interval of $\mathbf{P}^{\text{ad}}_{i}$. Fig. 2 gives an example for adjacent partition of v_1 with $V = \{v_1, v_2, v_3, v_4\}$ and $\mathcal{T} = [t_0,\ t_{11}]$. In the following we use $\mathbf{P}^{\text{ad}}_{V} = \{\mathbf{P}^{\text{ad}}_{1}, ..., \mathbf{P}^{\text{ad}}_{N}\}$ to represent the set of adjacent partitions of all the nodes in V. By definition, we can get that $\mathbf{P}^{\text{ad}}_{1,2} = \{t_0, t_1, t_2, t_6, t_{11}\}$, $\mathbf{P}^{\text{ad}}_{1,3} = \{t_0, t_9, t_{11}\}$ and $\mathbf{P}^{\text{ad}}_{1,4} = \{t_0, t_3, t_{10}, t_{11}\}$. By combining $\mathbf{P}^{\text{ad}}_{1,2}$, $\mathbf{P}^{\text{ad}}_{1,3}$ and $\mathbf{P}^{\text{ad}}_{1,4}$ and reorder the time points, we can get the adjacent partition of v_1: $\mathbf{P}^{\text{ad}}_{1} = \{t_0, t_1, t_2, t_3, t_6, t_9, t_{10}, t_{11}\}$.

Proposition 5.1: (*ET-law*) Given a feasible broadcast relay schedule $\mathbf{S}$, suppose node v_i is informed at time t' and is scheduled to transmit a packet at time t $(t \geq t')$, where t lives in an interval $[t_s,\ t_e)$ in v_i's adjacent partition. Since the nodes connected by v_i are not changed during $[t_s,\ t_e)$, when $t' \notin [t_s,\ t_e)$, the nodes that receive the packet from v_i are the same when v_i transmits at t_s and at t. Thus, if we change v_i's transmission time t to t_s in $\mathbf{S}$, $\mathbf{S}$ is still feasible. Then, feasible schedule $\mathbf{S}$ remains feasible if we change the transmission time t to its possible earliest time t_{earliest}:

$$t_{\text{earliest}} = \begin{cases} t' & \text{if } t' \in [t_s,\ t_e) \\ t_s & \text{if } t' \notin [t_s,\ t_e) \end{cases} \tag{10}$$

We say a schedule follows *Earliest transmission law* (or simply *ET-law*) if each relay node transmits its packet at its t_{earliest} and then we call each transmission an ET transmission.

Status partition. Given a broadcast schedule that follows ET-law, $\forall v_i \in V$, we call an interval $[t_s, t_e)$ $(t_s, t_e \in \mathcal{T})$ a *status interval* of v_i if its status cannot possibly be changed during $[t_s, t_e)$. We call a $\mathcal{T}$ partition a *status partition* of v_i (denoted by $\mathbf{P}^{\text{st}}_{i}$) if the partition is composed of v_i's status intervals. We use $\mathbf{P}^{\text{st}}_{V} = \{\mathbf{P}^{\text{st}}_{1}, ..., \mathbf{P}^{\text{st}}_{N}\}$ to represent the set of status partitions of all the nodes in V. Notice that given a TVEG, though each node has unique adjacent partition, it has infinite number of status partitions because its status interval can be continuously partitioned into a set of smaller intervals.

As mentioned previously, each node v_i's DTS is the combination of its adjacent partition and its status partition. During each interval of DTS, the set of nodes that v_i connects to and v_i's status are unchanged. Then TMEDB on continuous time can be transferred to TMEDB on DTS.

Definition 5.2: (*Discrete time set*) Suppose the broadcast schedule $\mathbf{S}$ follows ET-law, a discrete time partition of v_i is defined as the combination of its adjacent partition $\mathbf{P}^{\text{ad}}_{i}$ and one of its status partitions $\mathbf{P}^{\text{st}}_{i}$, *i.e.*,

$$\mathbf{P}^{\text{di}}_{i} \triangleq \mathbf{P}^{\text{ad}}_{i} \cup \mathbf{P}^{\text{st}}_{i}. \tag{11}$$

The DTS on V (or simply DTS), $\mathbf{D}_V$, is defined as a set of discrete time partitions for all the nodes in V, *i.e.*,

$$\mathbf{D}_V \triangleq \{\mathbf{P}^{\text{di}}_{1}, \mathbf{P}^{\text{di}}_{2}, ..., \mathbf{P}^{\text{di}}_{N}\}. \tag{12}$$

We say a schedule $\mathbf{S}$ is a schedule on DTS $\mathbf{D}_V$ if for any transmission $s = [v_i, t, w]$ in $\mathbf{S}$, the transmission time t is in v_i's discrete time partition $\mathbf{P}^{\text{di}}_{i}$.

Theorem 5.2: A TMEDB instance has feasible schedule iff it has a feasible schedule on DTS.

Proof: $\Leftarrow$: If a TMEDB instance has feasible schedule on DTS, then the instance must have feasible schedule on continuous time.

$\Rightarrow$: Suppose a TMEDB instance has feasible schedule, to prove it has a schedule on DTS, it is equivalent to prove it has a schedule following ET-law. For the sake of contradiction, we assume that there is no feasible schedule that follows ET-law. Among all feasible schedules, we denote the schedule with the maximum number of ET transmissions by $\mathbf{S}$. By the assumption, in $\mathbf{S}$, there exists a transmission $s_i = [r_i, t_i, w_i]$ that is not an ET transmission, where t_i lives in an interval $[t_s, t_e)$. According to Proposition 5.1, changing the time of s_i from t_i to t_s does not change the feasibility of the schedule, and the new schedule has

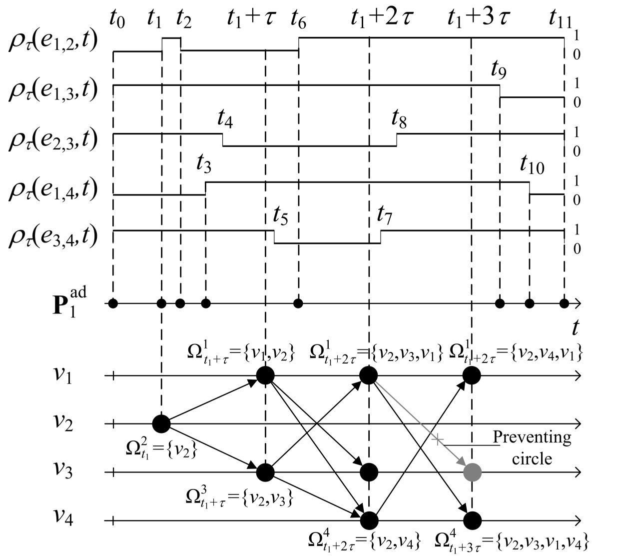


Figure 2. Discrete time points of v_1, v_3, v_4 triggered by v_2's starting time t_1.

more ET transmissions than $\mathbf{S}$ does. It contradicts with our assumption. ■

Let $L = \max\{|\mathbf{P}_1^{\mathrm{ad}}|, ..., |\mathbf{P}_N^{\mathrm{ad}}|\}$. Hence, there are totally $O(NL)$ points in $\mathbf{P}_V^{\mathrm{ad}}$. Because the length of each non-stop journey is $O(N)$, for each adjacent partition point, the number of time points on DTS it creates is $O(N^2)$, which implies that the total number of time points on DTS is $O(N^3L)$. Notice that in some real-life contact traces [12], transmission delay τ is much smaller than communication time among the nodes. In this case, we approximate τ by 0, and each adjacent partition time point only creates 1 DTS vector, which implies that the number of time points in DTS is $O(N^2L)$.

VI. Energy Efficient Broadcast Schemes

In Section V, we have proved that if we find an optimal solution for TMEDB on DTS, then this solution must also be an optimal solution on continuous time. Guided by this conclusion, in this section we design the schemes for TMEDB with both static channel (denoted by TMEDB-S) and fading channels (denoted by TMEDB-R). In Section VI-A, by constructing an auxiliary graph for TMEDB on DTS, we show that the existing polynomial time algorithm for minimum-energy multicast tree problem [3] can be used to provide $O(N^\epsilon)$ approximation ratio for TMEDB-S. In Section VI-B, we breakdown TMEDB-R into two sub-problems, called *broadcast backbone selection* and *optimal energy allocation*. The first sub-problem can be solved by the algorithm for static channel case. For the second sub-problem, we formalize it as a nonlinear programming problem, which can be solved by the existing methods [19].

A. Energy-efficient delay-constrained broadcast (EEDCB)

Recall that at time t for any pair of adjacent nodes v_i and v_j, the minimum cost from v_i to v_j is given by $w_{i,t}^j = N_0\gamma_{\mathrm{th}}/h_{i,j,t}$ (Equ. (2)). Given a node $v_i \in V$, suppose there are m_i nodes v_1, ..., v_{m_i} adjacent to v_i at time t with the minimum cost $w_{i,t}^1$, ..., $w_{i,t}^{m_i}$, respectively, where $w_{i,t}^k < w_{i,t}^{k+1}$ ($k = 1, 2, ..., m_i - 1$). Then, the cost set of v_i at time t can be partitioned into a set of cost intervals: $[w_{i,t}^1, w_{i,t}^2)$, ..., $[w_{i,t}^{m_i}, w_{\max}]$. We define $\mathcal{W}_{i,t}^{\mathrm{di}} = \{w_{i,t}^1$, ..., $w_{i,t}^{m_i}\}$ as the *discrete cost set* (*DCS*) of v_i at time t.

Property 6.1: Given node v_i's discrete cost set at time t: $w_{i,t}^1, ..., w_{i,t}^{m_i}$, (i) (Broadcast nature) if v_i broadcasts the packet with cost $w_{i,t}^k$ ($1 \le k \le m_i$), then each node v_l with $1 \le l \le k$ can be informed by v_i; (ii) if v_i can inform a set of nodes $\mathcal{D}$ with cost $w \in [w_k,\ w_{k+1})$, then v_i can inform $\mathcal{D}$ with cost w_k.

Proposition 6.1: The existence of a feasible schedule for TMEDB implies the existence of a feasible schedule *s.t.* the cost of each transmission is selected from the relay's DCS.

Proof: For the sake of contradiction, suppose there exists no such feasible schedule. Among all feasible schedules, let $\mathbf{S}$ be the feasible schedule that has the maximum number of transmissions, the cost of which is selected from its relay node's discrete cost set. In $\mathbf{S}$, there must exit a transmission, in which the sender v_i uses the cost w that is not in its discrete cost set $\mathcal{W}_{i,t}^{\mathrm{di}} = \{w_{i,t}^k | 1 \le k \le m_i\}$. Suppose $w \in (w_{i,t}^k, w_{i,t}^{k+1})$, according to Property 6.1 (ii), by changing the cost from w to w_{i_k}, we can get a new feasible schedule $\mathbf{S}'$, which has one more transmission satisfying the condition than $\mathbf{S}$, which contradicts with our assumption. ■

Guided by Proposition 6.1, to find the solution of TMEDB-S, we only need to find the solution of TMEDB in which the cost of each transmission is in the sender's DCS. Then, by building an auxiliary graph, TMEDB-S can be transferred to a previous problem called *minimum-energy multicast tree problem* (*MEMT*) [3], which is defined as follows: given a wireless network $\mathcal{M} = (\mathcal{U}, \mathcal{L})$, where each node has k power levels, a source node v_{s}, and a terminal set D ($D \subset \mathcal{U}$), the objective is to broadcast a packet from v_{s} to the nodes in D such that the total transmission energy of all involved nodes is minimized. In [3], the author has proposed an approximate algorithm with approximation ratio upper bound $O(N^\epsilon)$ and time complexity $O((k+1)^{\frac{1}{\epsilon}} N^{\frac{3}{\epsilon}})$. Below, we introduce the problem transferred by building an auxiliary graph.

Our goal for building auxiliary graph is to map TMEDB-S to MEMT in the auxiliary graph. We first build the virtual node set $V_{\mathrm{aux}} = \{u_{i,l} | 1 \le i \le N, 1 \le l \le h_i\}$, where h_i is the number time points in v_i's discrete time partition $\mathbf{P}_i^{\mathrm{di}}$ and $u_{i,l}$ corresponds to the l^{th} time point in $\mathbf{P}_i^{\mathrm{di}}$. Then, we need to add edges in the graph to reflect the transmission cost among these nodes. (i) For any pair of nodes $u_{i,l}$ and $u_{i,l+1}$, we build a 0-weight edge from $u_{i,l}$ to $u_{i,l+1}$. It means that if

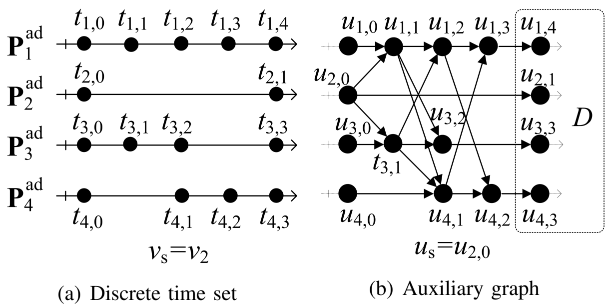


Figure 3. Auxiliary graph derived from a discrete time set.

v_i has received the packet at time $t_{i,l}$, it must have received the packet at time $t_{i,l+1}$. (ii) If v_i is adjacent to v_j at time $t_{i,l}$ ($t_{j,f} = t_{i,l} - \tau$) with minimum cost $w^k_{i,t_{i,l}}$, then we build a directed edge from $u_{i,l}$ to $u_{j,f}$ with cost $w^k_{i,t_{i,l}}$. It means that if v_i has received the packet at time $t_{i,l}$, v_i needs to use energy $w^k_{i,t_{i,l}}$ to inform v_j at this time. Then, TMEDB-S is equivalent to finding the minimum-energy multicast tree on the auxiliary graph $\mathcal{G}_{\text{aux}}(V_{\text{aux}}, E_{\text{aux}})$, where the terminal nodes are $D = \{u_{1,h_1}, u_{2,h_2}, ..., u_{N,h_N}\}$. Fig. 3 gives an example on this map: Fig. 3 (a) shows the discrete time set of v_1, v_2, v_3, and v_4: $\mathbf{D}_V = [\mathbf{P}_1^{\text{di}}, \mathbf{P}_2^{\text{di}}, \mathbf{P}_3^{\text{di}}, \mathbf{P}_4^{\text{di}}]$, where v_2 is the source node; Fig. 3 (b) shows the corresponding auxiliary graph, where $u_{2,0}$ is the source node and $D = \{u_{1,4}, u_{2,1}, u_{3,3}, u_{4,3}\}$ is the destination node set.

According to the complexity analysis in Section V, the number of nodes in $\mathcal{G}_{\text{aux}}$ is $O(N^2 L)$, and the size of each discrete cost set is $O(N)$. Finally, using the algorithm introduced in [3], we can get the solution with upper approximation ratio bound $O(N^\epsilon)$, and the time complexity of the solution is

$$O((N+1)^{\frac{1}{\epsilon}}(N^2 L)^{\frac{3}{\epsilon}}) = O(N^{\frac{7}{\epsilon}} L^{\frac{3}{\epsilon}}) \tag{13}$$

B. Fading-resistant EEDCB (FR-EEDCB)

We then discuss the TMEDB problem in the Rayleigh fading environment (TMEDB-R), in which the ED-function class is composed of a set of Rayleigh ED-functions (Equ. (5)). In fading environment, the signal power received by each node is random fluctuated and the probabilities of successful transmissions are always smaller than 1. Hence, in multi-hop transmissions, the energy used by each relay node determines the probability of successfully decoding the packet of not only its successor but also the nodes after the successor. Therefore, we cannot directly use the auxiliary graph introduced in Section VI-A to solve TMEDB-R. Recall that a solution includes relay nodes, their transmission times and energy levels. To solve the TMEDB-R problem, we then breakdown this problem to two sub-problems, namely *broadcast backbone selection*, which determines relay nodes $\mathbf{R} = [r_1, r_2, ..., r_n]$ and their transmission times $\mathbf{T} = [t_1, t_2, ..., t_n]$, and *optimal energy allocation*, which determines the energy level $\mathbf{W} = [w_1, w_2, ..., w_n]$ that each relay node uses for transmitting the packet.

Broadcast backbone selection. We simply use the broadcast algorithm for the static channel model (introduced in Section VI-A) to select relay nodes $\mathbf{R}$ and determine their transmission times $\mathbf{T}$. To build the auxiliary graph, we need to give each edge a weight, which is determined by the channel condition of the edge. Here, for any edge $e_{i,j}$, it has ED-function $\varphi_t^{e_{i,j}}(w) = 1 - \exp\left(-\frac{N_0 \gamma_{\text{th}}}{w d^{-\alpha}}\right)$ (Equ. (5)). When $\varphi_t^{e_{i,j}}(w_0) = \varepsilon$, we calculate the weight of this edge to be $w_0 = \frac{N_0 \gamma_{\text{th}}}{\ln(1/(1-\varepsilon)) d^{-\alpha}}$, which is the cost that can make the probability of failure transmission equal the acceptable error rate ε in single hop.

Optimal energy allocation. After broadcast backbone selection, relay nodes $\mathbf{R}$ and their transmission times $\mathbf{T}$ have been determined. For a schedule, what remains to solve is how much energy relay nodes should use for forwarding the packet, *i.e.*, $\mathbf{W} = [w_1, w_2, ..., w_n]$. Recall that the objective of TMEDB is to minimize the total energy consumption while guaranteeing that (1) the broadcast can be completed within the time constraint; and (2) the delivery ratio is higher than a threshold. Accordingly, we can formulate the optimal energy allocation problem as a nonlinear programming problem (NLP):

$$\min \sum_{k=1}^{n} w_k \tag{14}$$

$$\text{s.t.} \prod_{\substack{[r_k, t_k] \in [\mathbf{R}, \mathbf{T}], \\ \rho_\tau(e_{r_k, v_j}, t_k) = 1}} \varphi_{t_k}^{e_{r_k, v_j}}(w_k) \le \varepsilon, \ \forall\ v_j \in V \tag{15}$$

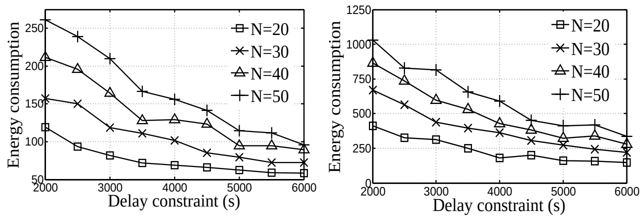
$$\prod_{\substack{[r_k, t_k] \in [\mathbf{R}, \mathbf{T}], \\ t_k \le t_j, \rho_\tau(e_{r_k, r_j}, t_k) = 1}} \varphi_{t_k}^{e_{r_k, r_j}}(w_k) \le \varepsilon, \ \forall\ r_j \in \mathbf{R} \tag{16}$$

$$w_{\min} \le w_k \le w_{\max}$$

where $w_1, w_2, ..., w_n$ are decision variables [20]. The first constraint (Formula (15)) means that all the nodes in V must have received the packet after all the relay nodes in $\mathbf{R}$ forward the packet. The second constraint (Formula (16)) means that for each relay node r_j, which forwards the packet at time t_j, it must have received the packet by time t_j.

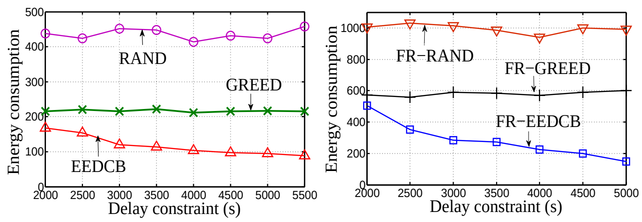
VII. Performance Evaluation

We conducted the trace-driven simulation based on the real trace from the Huggle project [12] for performance evaluation. We compared EEDCB and FR-EEDCB with the greedy algorithms GREED and FR-GREED, both of which select the node that can inform the largest number of nodes as relay node at each step, and also with the random algorithms RAND and FR-RAND, both of which randomly select an informed node as relay node at each step. In the



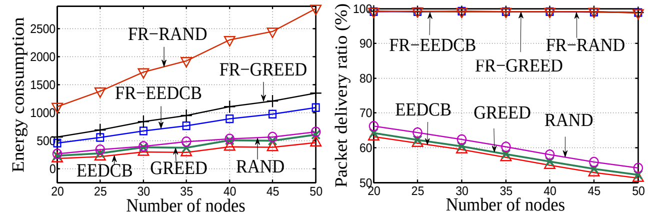
(a) EEDCB (b) FR-EEDCB

Figure 4. Delay-energy tradeoff of EEDCB and FR-EEDCB.



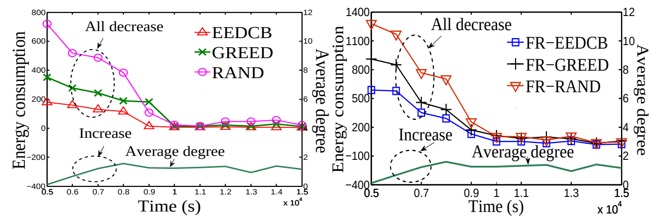
(a) Static channels (b) Fading channels

Figure 5. Delay-energy tradeoff of different algorithms.



(a) Energy consumption (b) Packet delivery ratio

Figure 6. Performance in the fading scenario.



(a) Static channels (b) Fading channels

Figure 7. Energy consumption and average node degree over time.

simulation, we randomly chose a source node for broadcast. In GREED and RAND, the transmission cost equals the minimum cost in the relay's discrete cost set, and in FR-GREED and FR-RAND, it is calculated by NLP (Equ. (14)-(17)). The parameters were set as follows [13]: the noise power density N_0 equals 4.32×10^{-21} W/HZ, the decoding threshold γ_{th} equals 25.9 dB, the date rate equals 1 Mbit/s, the path lose exponent α equals 2, and the acceptable error rate ϵ equals 0.01. We considered both static channels and fading channels using the Rayleigh fading model. Unless otherwise specified, the number of nodes was set to 20 and the delay constraint was set to 2000s. Each experiment lasted for approximately 17000s. We mainly measured the following metrics:

1) *Normalized energy consumption*. It is defined as the total energy consumption of all the nodes in a broadcast normalized by the decoding threshold γ_{th} [14].

2) *Packet delivery ratio*. It is defined as the percent of the nodes that have successfully received the broadcasted packet when every node transmits the packet once.

Fig. 4 (a) and Fig. 4 (b) show the normalized energy consumption versus the delay constraint of our methods with different number of nodes (N) with static channels and fading channels, respectively. We varied the delay constraint from 2000s to 6000s with 500s increase in each step. Both figures demonstrate that the energy consumption decreases as the delay constraint increases. The reason is that when the delay constraint is smaller, a node needs more energy to reach more nodes so that all nodes can receive the packet within the delay constraint. Also, both figures show that the normalized energy consumption increases as N increases since more nodes consumes more energy in total.

Fig. 5 (a) and Fig. 5 (b) compare the energy consumption of different algorithms with static channels and fading channels, respectively. We find that energy consumption follows EEDCB<GREED<RAND and FR-EEDCB<FR-GREED<FR-RAND. EEDCB and FR-EEDCB outperform the greedy algorithms and the random algorithms because EEDCB and FR-EEDCB try to achieve the global optimal solution to minimize energy consumption while the greedy algorithms always find the local optimal solution, and the random algorithms only randomly select a relay node at each step.

Fig. 6 (a) and Fig. 6 (b) show the energy consumption and packet delivery ratio of the different algorithms with different number of nodes in fading environment, respectively. We see that the energy consumption follows FR-RAND>FR-GREED>FR-EEDCB>RAND>GREED>EEDCB, and the packet delivery ratio follows FR-RAND≈ FR-GREED ≈ FR-EEDCB>RAND>GREED>EEDCB. EEDCB, GREED and RAND consume lower energy and also produce lower delivery ratio than others because they are only for non-fading environment, which requires lower energy in transmission. Thus, they generate lower packet delivery ratio in the fading environment. These algorithms fail to broadcast the packet to about 33%-37% nodes when the network has 20 nodes, and their packet delivery ratio rapidly decreases as the network grows. As the network grows, the broadcast tree becomes larger and the probability of link failure increases. By incorporating the fading model into these algorithms, FR-EEDCB, FR-GREED, and FR-RAND achieve nearly full delivery constantly for different network sizes. This result indicates the effectiveness of considering fading model in guaranteeing high packet delivery ratio in a fading environment.

We then study the effect of node degree in the network graph on the energy consumption for broadcast. We calculated the average degree of the 20 nodes in the network and their total normalized energy consumption every 500s from 5000s to 15000s. Fig. 7 (a) and Fig. 7 (b) show the total normalized energy consumption and average degree during [5000s, 15000s] with static channels and fading channels, respectively. The figures show that the average degree increases rapidly in [5000s, 8000s], and remains nearly constant afterwards. It is interesting to see that the energy consumption of all methods decreases rapidly during [5000s, 8000s], and stays nearly constant afterwards. This is because that as the average degree increases, each rely node is more likely to inform more nodes when it forwards the packet. Hence, both the size of broadcast backbone and the total number of transmissions decrease, thus reducing the total energy consumption.

VIII. Conclusion

In this paper, to build a model for studying dynamic wireless networks, we have introduced the notation of time-varying energy-demand graphs, which enhance the existing time-varying graphs by mapping each edge to an energy-demand function, which is improved from the existing time-varying graphs by mapping each edge to an energy-demand function. Based on this model, we have formulated a problem called time-varying minimum-energy delay-constrained broadcast problem (TMEDB). The objective of this problem is to find a relay allocation schedule to minimize the energy consumption of the broadcast within constraint time while guaranteeing that the delivery ratio is higher than a threshold. We have proved that this problem is NP-hard and $o(\log N)$ inapproximable. To facilitate designing an algorithm for this problem, we additionally have proved that TMEDB defined on continuous time is equivalent to TMEDB defined on discrete time sets (DTS). Guided by these property, we propose schemes for TMEDB for either static channels or fading channels. Finally, we evaluate the performance of these schemes. In our future work, we will take into account non-deterministic time-varying graphs and the interference among transmissions.

IX. Acknowledgement

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